Key

Math 4

Name _____

1-4 Solving Equations by Chunking/Substitution

Date _____

Review

Use algebraic reasoning to solve the following equations for the given variable. Show all work.
Use your calculator ONLY for equations involving *e* and *natural log*.
For the trigonometric equations, find solutions $0 \le \theta \le 2\pi$.

1. $2e^{3x} = 12$

$e^{3x} = 6$

$\ln 6 = 3x$

$\frac{\ln 6}{3} = x \approx 0.597$

2. $\ln(x+4) = -2$ (log$_e$)

$e^{-2} = x + 4$

$0.135 = x + 4$

$x \approx -3.865$

3. $x^2 + 8x + 12 = 0$

$(x+6)(x+2) = 0$

$x = -6$ or $x = -2$

4. $6x^2 + x - 5 = 0$

$(6x-5)(x+1) = 0$

$x = \frac{5}{6}, x = -1$

5. $4x^3 + 7x^2 = 2x^2 + 6x$

$4x^3 + 5x^2 - 6x = 0$

$x(4x^2 + 5x - 6) = 0$

$x(4x-3)(x+2) = 0$

$x = 0, x = \frac{3}{4}, x = -2$

6. $\ln(x^2) = \ln(x) + \ln(3)$

$\ln x^2 = \ln 3x$

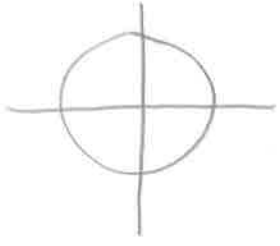
$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

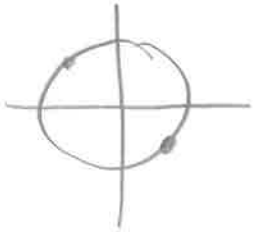
$x = 0$ or $x = 3$

7. $\sin(\theta) = \pi$



Undefined
sin only goes from -1 to 1

8. $\tan\theta = -1$



$135^\circ = \frac{3\pi}{4}$ radians

&

$315^\circ = \frac{7\pi}{4}$ radians

Solving by Substitution/Chunking Notes

$e^{2x} - e^x - 6 = 0$ $u = e^x$

$u^2 - u - 6 = 0$

$(u-3)(u+2) = 0$

$u = 3$ $u = -2$

$e^x = 3$ ~~$e^x = -2$~~

$\ln 3 = x$ ~~$\ln -2 = x$~~ Not possible

$x \approx 1.099$

$\frac{1}{x^2} + 8 = \frac{6}{x}$ $u = \frac{1}{x}$

$u^2 + 8 = 6u$

$u^2 - 6u + 8 = 0$

$(u-4)(u-2) = 0$

$u = 4$ $u = 2$

$\frac{1}{x} = 4$ $\frac{1}{x} = 2$

$1 = 4x$ $1 = 2x$

$\frac{1}{4} = x$ or $\frac{1}{2} = x$

$(x+3)^6 - 4(x+3)^3 - 21 = 0$

$u = (x+3)^3$

$u^2 - 4u - 21 = 0$

$(u-7)(u+3) = 0$

$u - 7 = 0$ $u + 3 = 0$

$u = 7$ $u = -3$

$(x+3)^3 = 7$ $(x+3)^3 = -3$

$x + 3 = \sqrt[3]{7}$ $x + 3 = \sqrt[3]{-3}$

$\sin^2 x - 4\sin x - 5 = 0$

$u = \sin x$

$u^2 - 4u - 5 = 0$

$(u-5)(u+1) = 0$

$u = 5$ $u = -1$

~~$\sin x = 5$~~ $\sin x = -1$

Not possible

$x = \frac{3\pi}{2}$ rad or $270°$

Practice

Use algebraic reasoning to solve the following equations for the given variable. Show all work.
Use your calculator ONLY for equations involving *e* and *natural log*.
For the trigonometric equations, find solutions $0 \le \theta \le 2\pi$.

1. $e^{2t} - 26e^{t} + 25 = 0$

$u = e^t$

$u^2 - 26u + 25 = 0$

$(u-25)(u-1) = 0$

$u = 25 \quad u = 1$

$e^t = 25 \quad e^t = 1$

$\ln 25 = t \quad \ln 1 = t$

$t \approx 3.219$ or $t = 0$

2. $2(\ln x)^2 + \ln x - 1 = 0$

$u = \ln x$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2} \quad u = -1$

$\ln x = \frac{1}{2} \quad \ln x = -1$

$e^{\frac{1}{2}} = x \quad e^{-1} = x$

$x \approx 1.649 \quad x \approx 0.368$

3. $y^{10} - 5y^6 + 4y^2 = 0$

$y^2(y^8 - 5y^4 + 4) = 0$

$u = y^4$

$y^2(u^2 - 5u + 4) = 0$

$y^2(u-4)(u-1) = 0$

$y = 0 \quad u = 4 \quad u = 1$

$y^4 = 4 \quad y^4 = 1$

$y = 0, \ y \approx 1.414, \ y = 1$

4. $x^6 + 9x^3 = 2x^3 + 8$

$x^6 + 7x^3 = 8$

$x^6 + 7x^3 - 8 = 0$

$u = x^3$

$u^2 + 7u - 8 = 0$

$(u+8)(u-1) = 0$

$u = -8 \quad u = 1$

$x^3 = -8 \quad x^3 = 1$

$x = -2 \quad x = 1$

5. $2\cos^2 x + \cos x - 1 = 0$

$u = \cos x$

$2u^2 + u - 1 = 0$

$(2u-1)(u+1) = 0$

$u = \frac{1}{2} \quad u = -1$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$x = \frac{\pi}{3} \quad x = \frac{5\pi}{3}$

6. $3(x+2)^2 - 7(x+2) + 4 = 0$

$u = x + 2$

$3u^2 - 7u + 4 = 0$

$(3u-4)(u-1) = 0$

$u = \frac{4}{3} \quad u = 1$

$x + 2 = \frac{4}{3} \quad x + 2 = 1$

$x = -\frac{2}{3}$ or $x = -1$

7. $3x^6 - 20x^3 = 32$

$3x^6 - 20x^3 - 32 = 0$

$u = x^3$

$3u^2 - 20u - 32 = 0$

$(3u + 4)(u - 8) = 0$

$u = -\frac{4}{3}$ $\quad u = 8$

$x^3 = -\frac{4}{3}$ $\quad x^3 = 8$

$x \approx -1.101$ $\quad x = 2$

8. $(\log_2 x)^2 - 10\log_2 x = -16$

$u = \log_2 x$

$u^2 - 10u + 16 = 0$

$(u - 8)(u - 2) = 0$

$u = 8$ $\quad u = 2$

$\log_2 x = 8$, $\log_2 x = 2$

$2^8 = x$ $\quad 2^2 = x$

$x = 256$ $\quad x = 4$

9. $2t^4 + 3t^2 = 2$

$u = t^2$

$2u^2 + 3u - 2 = 0$

$(2u - 1)(u + 2) = 0$

$u = \frac{1}{2}$ $\quad u = -2$

$t^2 = \frac{1}{2}$ $\quad t^2 = -2$ Not possible

$t \approx 0.707$

10. $4\sin^2 x = 4\sin x - 1$

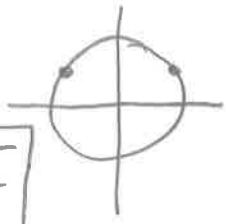
$u = \sin x$

$4u^2 - 4u + 1 = 0$

$(2u - 1)(2u - 1) = 0$

$u = \frac{1}{2}$

$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

11. $(8n - 7)^2 = 7 - 8n$

$(8n - 7)^2 = -(-7 + 8n)$

$(8n - 7)^2 = -(8n - 7)$

$u = 8n - 7$

$u^2 = -u$

$u^2 + u = 0$

$u(u + 1) = 0$

$u = 0$ $\quad u = -1$

$8n - 7 = 0$ $\quad 8n - 7 = -1$

$u = \frac{7}{8}, u = \frac{6}{8} = \frac{3}{4}$

12. $\frac{1}{(x-1)^2} - \frac{1}{(x-1)} - 2 = 0$

$u = \frac{1}{x-1}$

$u^2 - u - 2 = 0$

$(u - 2)(u + 1) = 0$

$u = 2$ $\quad u = -1$

$\frac{1}{x-1} = 2$ $\quad \frac{1}{x-1} = -1$

$2x - 2 = 1$ $\quad -x + 1 = 1$

$x = 3/2$ $\quad x = 0$